Exercise: Graph Theory Traversal and Shortest Paths

This document defines the lab for the ["Algorithms – Fundamentals (Java)" course @ Software University](https://softuni.bg).

Please submit your solutions (source code) to all below-described problems in [Judge](https://judge.softuni.bg).

1. Distance between Vertices

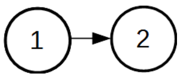
We are given a **directed graph**. We are given also a set of **pairs of vertices**. Find the **shortest distance between each pair** of vertices or **-1** if there is no path connecting them.

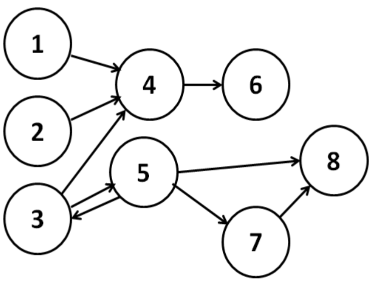
On the first line, you will get **N**, the number of vertices in the graph. On the second line, you will get P, the number of pairs between which to find the shortest distance.

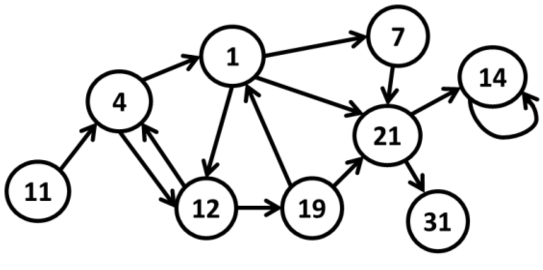
On the next **N,** lines will be the edges of the graph and on the next **P** lines, the pairs.

Examples

Input	Picture	Output

`2`
`2`
`1:2`
`2:`
`1-2`
`2-1`
		`{1, 2} -> 1`
		`{2, 1} -> -1`

`8`
`4`
`1:4`
`2:4`
`3:4 5`
`4:6`
`5:3 7 8`
`6:`
`7:8`
`8:`
`1-6`
`1-5`
`5-6`
`5-8`
		`{1, 6} -> 2`
		`{1, 5} -> -1`
		`{5, 6} -> 3`
		`{5, 8} -> 1`

9 8 11:4 4:12 1 1:12 21 7 7:21 12:4 19 19:1 21 21:14 31 14:14 31: 11-7 11-21 21-4 19-14 1-4 1-11 31-21 11-14		{11, 7} -> 3 {11, 21} -> 3 {21, 4} -> -1 {19, 14} -> 2 {1, 4} -> 2 {1, 11} -> -1 {31, 21} -> -1 {11, 14} -> 4

Hint

For each pair use **BFS** to find all paths from the source to the destination vertex.

2. Areas in Matrix

We are given a matrix of letters of size N * M. Two cells are neighbors if they share a common wall. Write a program to find the connected areas of neighbor cells holding the same letter. Display the **total number of areas** and the number of **areas for each alphabetical letter** (ordered by alphabetical order).

On the **first line** is given the **number of rows**.

Examples

Input	Picture	Output
6 aacccaac baaaaccc baabaccc bbdaaccc ccdccccc ccdccccc	a a c c c a a c b a a a a c c c b a a b a c c c b b d a a c c c c c d c c c c c c c d c c c c c	Areas: 8 Letter 'a' -> 2 Letter 'b' -> 2 Letter 'c' -> 3 Letter 'd' -> 1

3 aaa aaa aaa	a a a a a a a a a	Areas: 1 Letter 'a' -> 1
5 asssaadas adsdasdad sdsdadsas sdasdsdsa ssssasddd	a s s s a a d a s a d s d a s d a d s d s d a d s a s s d a s d s d s a s s s s a s d d d	Areas: 21 Letter 'a' -> 6 Letter 'd' -> 7 Letter 's' -> 8

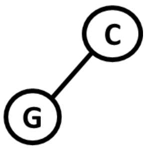
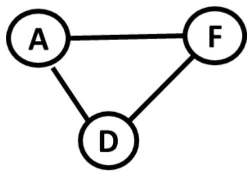
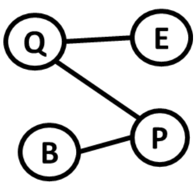
Hint

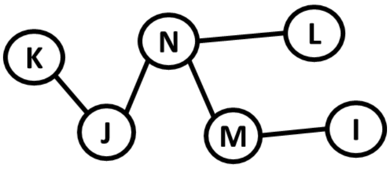
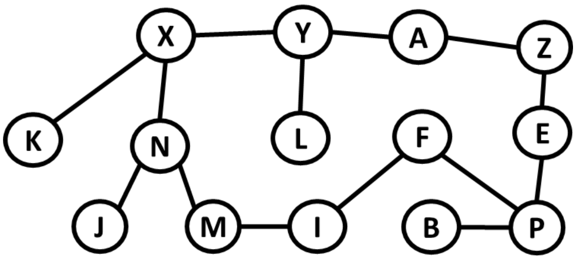
Initially mark all cells as **unvisited**. Start a **recursive DFS traversal** (or BFS) from each unvisited cell and mark all reached cells as visited. Each DFS traversal will find one of the **connected areas**.

3. Cycles in a Graph

Write a program to check whether an undirected graph is **acyclic** or holds any cycles. The input ends with the "**End**" line.

Examples

Input	Picture	Output
C-G End		Acyclic: Yes
A-F F-D D-A End		Acyclic: No
E-Q Q-P P-B End		Acyclic: Yes

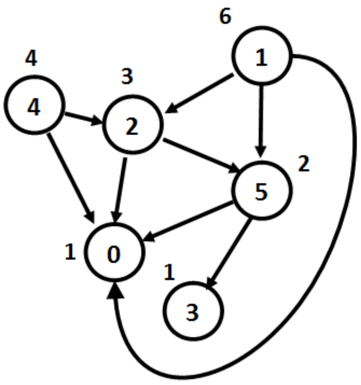
K-J J-N N-L N-M M-I End		Acyclic: Yes
K-X X-Y X-N N-J M-N A-Z B-P I-F A-Y Y-L M-I F-P Z-E P-E End		Acyclic: No

Hint

Modify the Topological Sorting algorithm (source removal or DFS-based).

4. Salaries

We have a **hierarchy** between the employees in a company. Employees can have one or several direct managers. People who **manage nobody** are called **regular employees** and their salaries are **1**. People who manage at least one employee are called **managers**. Each manager takes a **salary** that is equal to the **sum of the salaries of their directly managed employees**. Managers cannot manage directly or indirectly (transitively) themselves. Some employees might have no manager (like the big boss). See a sample hierarchy in a company along with the salaries computed following the above-described rule:



In the above example, employees 0 and 3 are regular employees and take salary 1. All others are managers and take the sum of the salaries of their directly managed employees. For example, manager 1 takes salary 3 + 2 + 1 = 6 (sum of the salaries of employees 2, 5 and 0). In the above example employees, 4 and 1 have no manager.

If we have **N** employees, they will be indexed from 0 to N – 1. For each employee, you'll be given a string with N symbols. The symbol at given index **i**, either **'Y' or 'N'**, shows whether the current employee is a direct manager of employee **i**.

Hint: find the node with no parent and start a **DFS traversal** from it to calculate the salaries on the tree recursively.

Input

- The input data should be read from the console.
- On the first line, you'll be given an integer N.
- On the next N lines, you'll be given strings with N symbols (either 'Y' or 'N').
- The input data will always be valid and in the format described. There is no need to check it explicitly.

Output

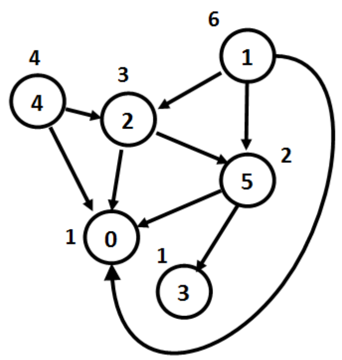
- The output should be printed on the console. It should consist of one line.
- On the only output line print the sum of the salaries of all employees.

Constraints

- N will be an integer in the range [1 … 50].
- For each i-th line, the i-th symbol will be 'N'.
- If employee A is the manager of employee B, B will not be a manager of A.
- Allowed working time for your program: 0.1 seconds. Allowed memory: 16 MB.

Examples

Input	Output	Comments
1 N	1	Only 1 employee with salary 1.
4 NNYN NNYN NNNN NYYN	5	We have 4 employees. 0, 1, and 3 are managers of 2. 3 is also a manager of 1. Therefore: salary(2) = 1 salary(0) = salary(2) = 1 salary(1) = salary(2) = 1 salary(3) = salary(2) + salary(1) = 2

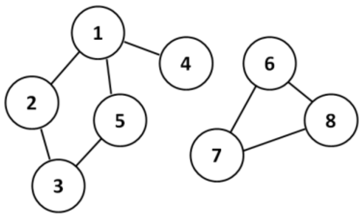
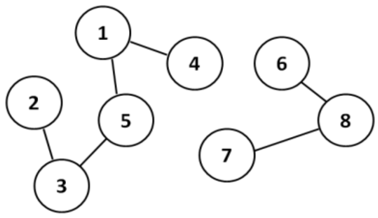
6 NNNNNN YNYNNY YNNNNY NNNNNN YNYNNN YNNYNN	17	

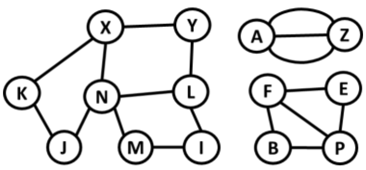
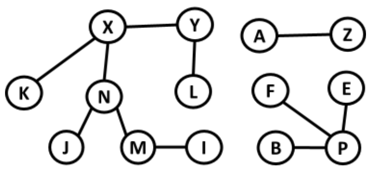
5. Break Cycles

You are given an **undirected multi-graph**. Remove a minimal number of edges to **make the graph acyclic** (to break all cycles). As a result, print the number of edges removed and the removed edges. If several edges can be removed to break a certain cycle, remove the smallest of them in alphabetical order (smallest start vertex in alphabetical order and smallest end vertex in alphabetical order).

Examples

Input	Picture	Output	Picture After Removal

`1 -> 2 5 4`
`2 -> 1 3`
`3 -> 2 5`
`4 -> 1`
`5 -> 1 3`
`6 -> 7 8`
`7 -> 6 8`
`8 -> 6 7`
		`Edges to remove: 2`
		`1 - 2`
		`6 - 7`
			

`K -> X J`
`J -> K N`
`N -> J X L M`
`X -> K N Y`
`M -> N I`
`Y -> X L`
`L -> N I Y`
`I -> M L`
`A -> Z Z Z`
`Z -> A A A`
`F -> E B P`
`E -> F P`
`P -> B F E`
		`Edgeds to remove: 7`
		`A - Z`
		`A - Z`
		`B - F`
		`E - F`
		`I - L`
		`J - K`
		`L - N`
			

B -> F P			

Hint

- Enumerate edges {**s**, **e**} in alphabetical order. For each edge {**s**, **e**} check whether it closes a cycle. If yes - remove it.
    - To check whether an edge {**s**, **e**} closes a cycle, temporarily remove the edge {s, e} and then try to find a path from **s** to **e** using DFS or BFS.

6. Road Reconstruction

You have to rebuild some roads in your city. Write a program that finds all the roads without which **buildings** in the city will become **unreachable**. You will receive how many **buildings** the town has on the first line, then you will receive the number of **streets** and finally, for **each street,** you will receive which **buildings it connects**. Find all the streets that are important and **cannot be removed** and print them as shown in the examples.

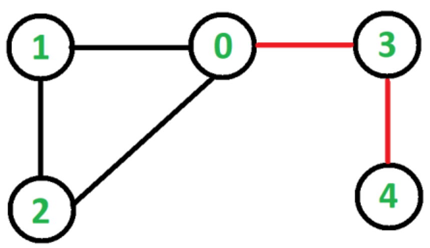
Input

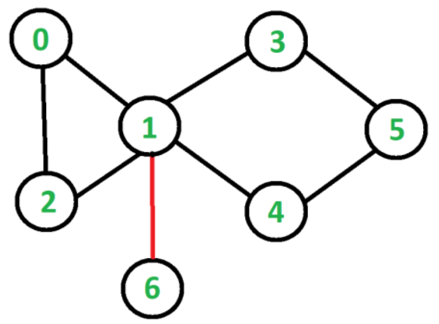
- On the first line, you will receive the **amount** of the **buildings.**
- On the second line, you will receive the **amount** of the **streets** (**n**).
- On the next **"n"** lines you will receive which **buildings** each **street connects.**

Output

- On the first line print: **`"Important streets:"`**
- On the next lines (if any) print the street in the format: **`"{firstBuilding} {secondBuilding}"`**
- The **order** of the output does **not matter** as long as you print all of the important streets.

Examples

Input	Output
5 5 1 - 0 0 - 2 2 - 1 0 - 3 3 - 4	Important streets: 3 4 0 3 
7 8 0 - 1 1 - 2 2 - 0 1 - 3 1 - 4 1 - 6	Important streets: 1 6

3 - 5 4 - 5	

12. The Matrix

You are given a matrix (2D array) of lowercase alphanumeric characters (**`a-z`**, **`0-9`**), a starting position – defined by a start row **`startRow`** and a start column **`startCol`** – and a filling symbol **`fillChar`**. Let's call the symbol originally at **`startRow`** and **`startCol`** the **`startChar`**. Write a program, which, starting from the symbol at **`startRow`** and **`startCol`**, changes to **`fillChar`** every symbol in the matrix which:

- is equal to **`startChar`** AND
- can be reached from **`startChar`** by going up (**`row - 1`**), down (**`row + 1`**), left (**`col - 1`**) and right (**`col + 1`**) and "stepping" ONLY on symbols equal **`startChar`**

So, you basically start from **`startRow`** and **`startCol`** and can move either by changing the row OR column (not both at once, i.e. you can't go diagonally) by **1** and can only go to positions that have the **`startChar`** written on them. Once you find all those positions, you change them to **`fillChar`**.

In other words, you need to implement something like the Fill tool in MS Paint, but for a 2D char array instead of a bitmap.

Input

On the first line, two integers will be entered – the number **R** of rows and number **C** of columns.

On each of the next **R** lines, **C** characters separated by single spaces will be entered – the symbols of the **R**th row of the matrix, starting from the **`0`**th column and ending at the **`C-1`** column.

On the next line, a single character – the **`fillChar`** – will be entered.

On the last line, two integers – **`startRow`** and **`startCol`** – separated by a single space, will be entered.

Output

The output should consist of R lines, each consisting of exactly C characters, **NOT SEPARATED** by spaces, representing the matrix after the fill operation has been finished.

Constraints

`0 < R, C < 20`
`0 <= startRow < R`
`0 <= startCol < C`

All symbols in the input matrix will be lowercase alphanumerics (**`a-z`**, **`0-9`**). The **`fillChar`** will also be alphanumeric and lowercase.

The total running time of your program should be no more than **`0.1s.`**

The total memory allowed for use by your program is **`5MB.`**

Examples

Input	Output
5 3 a a a a a a a b a a b a a b a x 0 0	xxx xxx xbx xbx xbx
5 3 a a a a a a a b a a b a a b a x 2 1	aaa aaa axa axa axa
5 6 o o 1 1 o o o 1 o o 1 o 1 o o o o 1 o 1 o o 1 o o o 1 1 o o 3 2 1	oo11oo o1331o 133331 o1331o oo11oo
5 6 o o o o o o o o o 1 o o o o 1 o 1 1 o 1 1 w 1 o 1 o o o o o z 4 1	oooooo ooo1oo oo1o11 o11w1z 1zzzzz
5 6 o 1 o o 1 o o 1 o o 1 o	z1oo1z z1oo1z z1111z

o 1 1 1 1 o o 1 o w 1 o o o o o o o z 4 0	z1zw1z zzzzzz

Hints

For some of the tests, you can solve the problem with a naive approach, however, a complete solution can be obtained by using **Stack**, **Queue**, **DFS,** or **BFS.**